University of Puerto Rico at Rio Piedras
Department of Mathematics
Qualifying Exam
MATE 6681: Higher Level Programming Languages

February 14, 2012

Name	Student ID

Instructions: Solve **exactly three** of the following five exercises. Please write clearly and show all your work to get credit. Good luck!

Exercise 1 (100 points) Show that the set

$$\{emptylist\} \cup char \cup (char \cup char) \cup (char \cup char \cup char) \cup ...$$

is the smallest solution to the set equation

$$CharList = \{emptylist\} \cup char \times CharList$$

Exercise 2 (100 points) Prove that the following program is correct:

```
{ n > 0 }
count := n;
sum := 0;
while count <> 0 do
    sum := sum + count;
    count := count - 1;
od
{sum = 1 + 2 +...+ n}
```

Exercise 3 (100 points) Let A be an array of n real numbers. Write recursive functions in:

a) C,

b) Haskell, and

c) Scheme

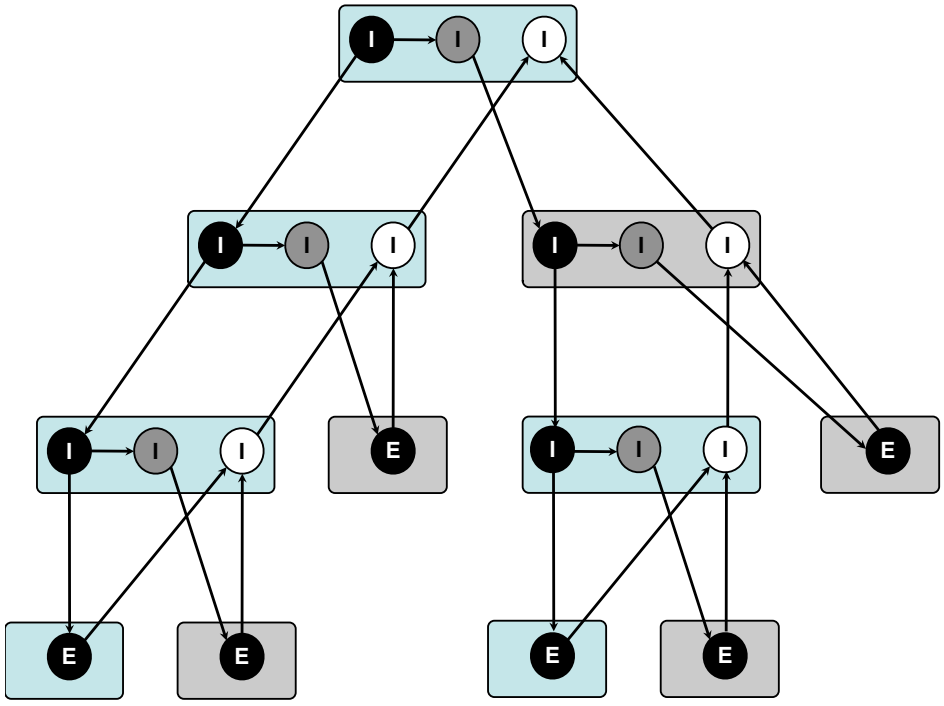
to compute the minimum element in A.

Exercise 4 (100 points) The following figure represents the direct acyclic graph for an instance of a threaded computation. Assuming that internal nodes (marked with an I) take 1 unit time and external nodes (marked with an E) take 4 unit times, determine:

a) *work*,

b) *span*, and

c) maximum *parallelism*.



Exercise 5 (100 points) Consider the following grammar in BNF:

$expr \rightarrow expr + term \mid term$
$term \rightarrow term * factor \mid factor$
$factor \rightarrow (\ expr\ ) \mid number$
$number \rightarrow number\ \ digit \mid digit$
$digit \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$

a) Rewrite the given grammar either in BNF **or** EBNF so that at most one unary minus is allowed in each expression, and it must come at the beginning of an expression, so $-2 + 3$ is legal (and equals 1) and $-2 + (-3)$ is legal, but $-2 + -3$ is not.

b) Draw a parse tree and an abstract syntax tree for the arithmetic expression:

$$3 * (4 + 5) * (6 + 7)$$